Chapter 2

The Big Three: Acceleration, Distance, and Time

In This Chapter

- Thinking about displacement
- Checking out speed
- Remembering acceleration

Being able to connect displacement, speed, and acceleration is fundamental to working with physics. These things concern people every day, and physics has made an organized study of them.

Problems that connect displacement, speed, and acceleration are all about understanding movement, and that's the topic of this chapter — putting numbers into the discussion. You'll often find physics problems about cars starting and stopping, horses racing, and rocket ships zooming back and forth. And after you finish this chapter, you'll be a real pro at solving them.

From Point A to B: Displacement

Displacement occurs when something moves from here to there. For example, suppose that you have a ball at the zero position, as in Figure 2-1A.

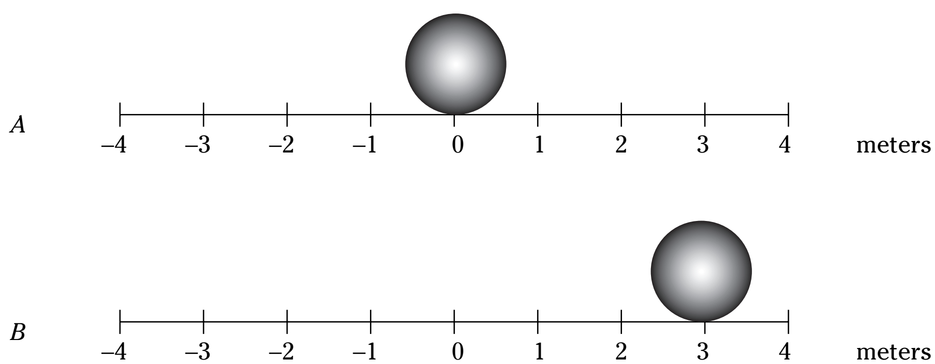

Figure 2-1: A moving ball.

Now suppose that the ball rolls over to a new point, 3 meters to the right, as you see in Figure 2-1B. The ball is at a new location, so there's been displacement. In this case, the displacement is just 3 meters to the right. In physics terms, you'll often see displacement referred to as the variable s. In this case, $s = +3$ meters.

Like any other measurement in physics, displacement is always expressed in units, usually centimeters or meters, as in this example. Of course, you also can use kilometers, inches, feet, miles, or even *light years* (the distance light travels in one year — 5,865,696,000,000 miles).

The following example question focuses on displacement.

EXAMPLE

Q. You've taken the pioneers' advice to "Go West." You started in New York City and went west 10 miles the first day, 14 miles the next day, and then back east 9 miles on the third day. What is your displacement from New York City after three days?

A. s = 15 miles west of New York City

1. You first went west 10 miles, so at the end of the first day, your displacement was 10 miles west.
2. Next, you went west 14 days, putting your displacement at 10 + 14 miles = 24 miles west of New York City.
3. Finally, you traveled 9 miles east, leaving you at 24 – 9 = 15 miles west of New York City. So s = 15 miles west of New York City.

1. Suppose that the ball in Figure 2-1 now moves 1 meter to the right. What is its new displacement from the origin, 0?

Solve It

2. Suppose that the ball in Figure 2-1, which started 4 meters to the right of the origin, moves 6 meters to the left. What is its new displacement from the origin — in inches?

Solve It

Reading That Speedometer

In physics terms, what is speed? It's the same as the conventional idea of speed: *Speed* is displacement divided by time.

For example, if you went a displacement s in a time t, then your speed, v, is determined as follows:

$$v = \frac{s}{t}$$

Technically speaking, speed is the change in position divided by the change in time, so you also can represent it like this if, for example, you're moving along the x axis:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_o}$$

Q. Suppose that you want to drive from New York City to Los Angeles to visit your uncle's family, a distance of about 2781 miles. The trip takes you four days. What was your speed in miles per hour?

A. $v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_o}{t_f - t_o} = 28.97 \text{ miles per hour}$

1. Start by figuring out your speed (the distance traveled divided by the time taken to travel that distance):

   $$\frac{2781 \text{ miles}}{4 \text{ days}} = 695.25$$

2. Okay, the speed is 695.25, but 695.25 *what?* This solution divides miles by days, so it's 695.25 miles per day — not exactly a standard unit of measurement. So what is that in miles per hour? To determine that, you cancel "days" out of this equation and put in "hours." Because 24 hours are in a day, you can multiply as follows (note that "days" cancel out, leaving miles over hours, or miles per hour):

   $$\frac{2781 \text{ miles}}{4 \cancel{\text{days}}} \times \frac{1 \cancel{\text{day}}}{24 \text{ hours}} = 28.97 \text{ miles per hour}$$

So your speed was 28.97 miles per hour. That's your average speed, averaged over both day and night.

3. Suppose that you used your new SpeedPass to get you through the tollbooths at both ends of your trip, which was 90 miles on the turnpike and took you 1 hour and 15 minutes. On your return home, you're surprised to find a traffic ticket for speeding in the mail. How fast did you go, on average, between the tollbooths? Was the turnpike authority justified in sending you a ticket, given that the speed limit was 65 mph?

Solve It

4. Suppose that you and a friend are determined to find out whose car is faster. You both start your trips in Chicago. Driving nonstop, you reach Los Angeles — a distance of 2018 miles — in 1.29 days, and your friend, also driving nonstop, reaches Miami — a distance of 1380 miles — in 0.89 days. Whose car was faster?

Solve It

Putting Pedal to Metal: Acceleration

In physics terms, *acceleration* is the amount by which your speed changes in a given amount of time. In terms of equations, it works like this:

$$a = \frac{\Delta v}{\Delta t}$$

Given initial and final velocities, v_o and v_f, and initial and final times over which your speed changed, t_o and t_f, you can also write the equation like this:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o}$$

To get the units of acceleration, you divide speed by time as follows:

$$a = \frac{v_f - v_o}{t_f - t_o} = \frac{\text{distance/time}}{\text{time}} = \frac{\text{distance}}{\text{time}^2}$$

Distance over time squared? Don't let that throw you. You end up with time squared in the denominator just because it's velocity divided by time — that's something you get used to when solving physics problems. In other words, acceleration is the *rate* at which your speed changes because rates have time in the denominator.

So for acceleration, you can expect to see units of meters per second^2, or centimeters per second^2, or miles per second^2, or feet per second^2, or even kilometers per hour^2.

Q. Suppose that you're driving at 75 miles an hour and suddenly see red flashing lights in the rearview mirror. "Great," you think, and you pull over, taking 20 seconds to come to a stop. You could calculate how quickly you decelerated as you were pulled over (information about your law-abiding tendencies that, no doubt, would impress the officer). So just how fast did you decelerate, in cm/sec^2?

A. $a = \dfrac{\Delta v}{\Delta t} = \dfrac{3350 \text{ cm/sec}}{20 \text{ seconds}} = 168 \text{ cm/sec}^2$

1. First convert to miles per second:

$$\frac{75 \text{ miles}}{\cancel{\text{hour}}} \times \frac{1 \cancel{\text{hour}}}{60 \cancel{\text{minutes}}} \times \frac{1 \cancel{\text{minute}}}{60 \text{ seconds}} = .0208 = 2.08 \times 10^{-2} \text{ miles per second}$$

2. Convert from miles per second to inches per second:

$$\frac{2.08 \times 10^{-2} \cancel{\text{miles}}}{\text{second}} \times \frac{5280 \cancel{\text{feet}}}{1 \cancel{\text{mile}}} \times \frac{12 \text{ inches}}{1 \cancel{\text{foot}}} = \frac{1318 \text{ in}}{\text{second}}$$

3. Your speed was 1318 inches per second. What's that in centimeters per second?

$$\frac{2.08 \times 10^{-2} \cancel{\text{miles}}}{\text{second}} \times \frac{5280 \cancel{\text{feet}}}{1 \cancel{\text{mile}}} \times \frac{12 \cancel{\text{inches}}}{1 \cancel{\text{foot}}} \times \frac{2.54 \text{ centimeters}}{\cancel{\text{inch}}} = \frac{3350 \text{ cm}}{\text{second}}$$

4. What was your acceleration? That calculation looks like this:

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_o}{t_f - t_o} = \frac{0 - 3350 \text{ cm/second}}{20 \text{ seconds}} = -168 \text{ cm/sec}^2$$

In other words, –168 cm/sec^2, *not* +168 cm/sec^2. There's a big difference between positive and negative in terms of solving physics problems — and in terms of law enforcement. If you accelerated at +168 cm/sec^2 instead of accelerating at –168 cm/sec^2, you'd end up going 150 miles per hour at the end of 20 seconds, not 0 miles per hour.

In other words, the *sign* of the acceleration tells you *how* the speed is changing. A positive acceleration means that the speed is increasing in the positive direction, and a negative acceleration (also known as *deceleration*) tells you that the speed is increasing in the negative direction.

5. A rocket ship is going to land on the moon in exactly 2 hours. There's only one problem: It's going 17,000 miles an hour. What does its deceleration need to be, in miles per second^2, in order to land on the moon safely at 0 miles per hour?

Solve It

6. You're stopped at a red light when you see a monster SUV careening toward you. In a lightning calculation, you determine you have 0.8 seconds before it hits you and that you must be going at least 1.0 miles an hour forward at that time to avoid the SUV. What must your acceleration be, in miles per hour^2? Can you avoid the SUV?

Solve It

7. A bullet comes to rest in a block of wood in 1.0×10^{-2} seconds, with an acceleration of -8.0×10^4 meters per second^2. What was its original speed, in meters per second?

Solve It

8. The light turns red, and you come to a screeching halt. Checking your stopwatch, you see that you stopped in 4.5 seconds. Your deceleration was 1.23×10^{-3} miles per second^2. What was your original speed in miles per hour?

Solve It

Connecting Acceleration, Time, and Displacement

You know that you can relate speed with displacement and time. And you know that you can relate speed and time to get acceleration. You also can relate displacement with acceleration and time:

$$s = \frac{1}{2} a \left(t_f - t_o\right)^2$$

If you don't start off at zero speed, you use this equation:

$$s = v_o\left(t_f - t_o\right) + \frac{1}{2} a \left(t_f - t_o\right)^2$$

Q. You climb into your drag racer, waving nonchalantly at the cheering crowd. You look down the quarter-mile track, and suddenly the flag goes down. You're off, getting a tremendous kick from behind as the car accelerates quickly. A brief 5.5 seconds later, you pass the end of the course and pop the chute.

You know the distance you went: 0.25 miles, or about 402 meters. And you know the time it took: 5.5 seconds. So just how hard was the kick you got — the acceleration — when you blasted down the track?

A. 26.6 meters/second^2

1. You know that

$$s = \frac{1}{2} a\ t^2$$

You can rearrange this equation with a little algebra (just divide both sides by t^2 and multiply by 2) to get

$$a = \frac{2s}{t^2}$$

2. Plugging in the numbers, you get

$$a = \frac{2s}{t^2} = \frac{2\ (402\ \text{meters})}{(5.5\ \text{seconds})^2} = 26.6\ \text{meters/second}^2$$

What's 26.6 meters/second^2 in more understandable terms? The acceleration due to gravity, *g,* is 9.8 meters/second^2, so this is about 2.7 g. And that's quite a kick.

9. The light turns green, and you accelerate at 10 meters per $second^2$. After 5 seconds, how far have you traveled?

Solve It

10. A stone drops under the influence of gravity, 9.8 meters per $second^2$. How far does it drop in 12 seconds?

Solve It

11. A car is going 60 miles per hour and accelerating at 10 miles per $hour^2$. How far does it go in 1 hour?

Solve It

12. A motorcycle is going 60 miles per hour, and decelerating at 60 miles per $hour^2$. How far does it go in 1 hour?

Solve It

13. An eagle starts at a speed of 50 meters per second and, decelerating at 10 meters per second^2, comes to rest on a peak 5 seconds later. How far is the peak from the eagle's original position?

Solve It

14. A trailer breaks loose from its truck on a steep incline. If the truck was moving uphill at 20 meters per second when the trailer broke loose, and the trailer accelerates down the hill at 10.0 meters per second^2, how far downhill does the trailer go after 10 seconds?

Solve It

15. A block of wood is shooting down a track at 10 meters per second and is slowing down because of friction. If it comes to rest in 20 seconds and 100 meters, what is its deceleration, in meters per second^2?

Solve It

16. A minivan puts on the brakes and comes to a stop in 12 seconds. If it took 200 meters to stop, and decelerates at 10 meters per second^2, how fast was it originally going, in meters per second?

Solve It

Connecting Speed, Acceleration, and Displacement

Suppose you have a drag racer whose acceleration is 26.6 meters/second^2, and its final speed was 146.3 meters per second. What is the total distance traveled?

This scenario sets you up to use one of the important equations of motion:

$$v_f^2 - v_o^2 = 2as = 2a\,(x_f - x_o)$$

This is the equation you use to relate speed, acceleration, and distance.

EXAMPLE

Q. A drag racer's acceleration is 26.6 meters/second^2, and at the end of the race, its final speed is 146.3 meters per second. What is the total distance the drag racer traveled?

A. $s = \frac{1}{2a} v_f^2 = \frac{1}{2(26.6)} (146.3)^2 = 409$ meters

1. To solve this problem, you need to relate speed, acceleration, and distance, so you start with this equation:

$$v_f^2 - v_o^2 = 2as = 2a\,(x_f - x_o)$$

2. In this scenario, v_o is 0, which makes this equation simpler:

$$v_f^2 = 2as$$

3. Solve for s:

$$s = \frac{1}{2a} v_f^2$$

4. Plug in the numbers:

$$s = \frac{1}{2a} v_f^2 = \frac{1}{2(26.6)} (146.3)^2 = 409 \text{ meters}$$

So the answer is 409 meters, about a quarter of a mile — standard for a drag racing track.

17. A bullet is accelerated over a meter-long rifle barrel at an acceleration of 400,000 meters per second^2. What is its final speed?

Solve It

18. A car starts from rest and is accelerated at 5.0 meters per second^2. What is its speed 500 meters later?

Solve It

19. A rocket is launched at an acceleration of 100 meters per second^2. After 100 kilometers, what is its speed in meters per second?

Solve It

20. A motorcycle is going 40 meters per second and is accelerated at 6 meters per second^2. What is its speed after 200 meters?

Solve It